

Neutrinos and Lepton-Flavour-Violation *

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I give a sketchy overview on aspects related to the lepton flavour sector in the standard model and its possible extensions.

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1. Introduction

The standard model (SM) of particle physics successfully describes a variety of phenomena in terms of a few fundamental quantities. Besides the gauge coupling constants associated to the fundamental interactions, and the scalar sector responsible for spontaneous electroweak symmetry breaking, the major part of adjustable parameters are related to the fermion sector describing the masses and mixings of quark and lepton flavours.

The experimental observation of neutrino oscillations implies that neutrinos have (tiny) masses, and the SM – which in its original version is restricted to massless neutrinos – has to be extended. The most popular approaches are based on “see-saw” scenarios, where neutrino masses are suppressed by a large scale which originates from integrating out heavy particles. In particular, heavy Majorana neutrinos appear naturally when embedding the SM in grand unified theories (GUTs). The violation of lepton number L at the Majorana mass scale allows to explain the baryon asymmetry in the universe by leptogenesis, where an L -asymmetry is built by out-of-equilibrium decays of Majorana neutrinos, which is subsequently transformed into a baryon asymmetry by electroweak sphaleron processes. The required CP-violation enters through interference effects of tree-level and loop diagrams and depends on the phases in the Majorana sector.

While the above phenomena relate the lepton flavour sector to high-energy scales and physics near the GUT scale, low-energy observables from

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lepton-flavour violating (LFV) processes are very sensitive to new physics (NP) at the TeV scale. For instance, in the minimally extended SM (see below), the decay $\mu \rightarrow e\gamma$ has a tiny branching fraction, $\mathcal{B}[\mu \rightarrow e\gamma]_{\text{SM}} \sim 10^{-54}$ compared to expectations from generic NP models which may be close to the present (foreseen) experimental reach, $\mathcal{B}[\mu \rightarrow e\gamma]_{\text{exp.}} < 10^{-11(13)}$. For more comprehensive discussions and more references to the original literature, I refer to the recent reviews [1, 2, 3, 4, 5, 6].

2. Massive neutrinos and lepton-flavour mixing

2.1. Extending the SM lepton sector

In the (original) SM, only charged leptons obtain a mass from the Yukawa coupling to the Higgs field H ,

$$(Y_E)^{ij} (\bar{L}^i H) E_R^j + \text{h.c.} \quad (2.1)$$

Here L and E_R denote the three families of left-handed lepton doublets and right-handed lepton singlets, respectively, and Y_E is the corresponding Yukawa coupling matrix. The Lagrangian describes massless neutrinos with *individual* lepton flavour (L_e, L_μ, L_τ) being conserved (i.e. no mixing). Adding right-handed Dirac neutrinos ν_R , and enforcing L -conservation, one obtains the analogous situation to the quark sector (i.e. CKM-like mixing). In view of the observed qualitative differences between the lepton and the quark sector, such a scenario seems to be less appealing.

Considering, instead, the SM as an effective theory, one can obtain neutrino masses in a minimal way by including the dimension-5 operator

$$\frac{(g_\nu)^{ij}}{\Lambda_L} (\bar{L}^i \tilde{H})(\tilde{H}^\dagger L^j)^c + \text{h.c.} \quad (2.2)$$

which violates lepton number, with the associated high-energy scale denoted as Λ_L . The mismatch between the diagonalization of the Yukawa matrix Y_E and of the new flavour matrix $g_\nu = g_\nu^T$ yields the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix.

Another option is to introduce right-handed Majorana neutrinos ν_R via

$$(Y_\nu)^{ij} (\bar{L}^i \tilde{H} \nu_R^j) + \frac{1}{2} M^{ij} (\nu_R^T)^i (\nu_R)^j + \text{h.c.} \quad (2.3)$$

In this case, lepton number is violated by the Majorana mass term M , and one can reproduce the dim-5 term in (2.2) by integrating out ν_R ,

$$\frac{g_\nu}{\Lambda_L} = Y_\nu (M)^{-1} Y_\nu^T, \quad (2.4)$$

Table 1. Parameter counting for the minimally extended SM.

Quantity	Symbol	Moduli	Phases
charged Yukawa matrix	Y_E	9	9
dim-5 neutrino matrix	$g_\nu = g_\nu^T$	6	6
flavour symmetry group	$U(3)_L \times U(3)_{E_R}$	-6	-12
physical parameters:	masses m_ℓ^i, m_ν^i	6	
	angles & phases	3	3

realizing the so-called type-I see-saw mechanism. Alternative mechanisms introduce heavy scalar triplets (type-II see saw), or heavy fermion triplets (type-III see saw).

In the minimally extended SM (2.2), the parameter counting for the lepton-flavour sector follows from the symmetries broken by the Yukawa sector [7], see Table 1. The low-energy neutrino-mixing parameters in the PMNS matrix are thus given by 3 angles, 1 Dirac phase and 2 Majorana phases. As has been pointed out (e.g. [8]), see-saw scenarios in general contain additional flavour parameters, which may not be directly accessible at low energies. For example, with 3 heavy Majorana neutrinos, the high-energy theory contains 3 additional angles and 3 additional phases, which can be parametrized in terms of an orthogonal complex matrix [9], such that in a basis where the charged-lepton Yukawa matrix Y_E and the Majorana mass matrix M is diagonal, one has

$$Y_\nu^T = \text{diag}[\sqrt{M_\nu}] R \text{diag}[\sqrt{m_\nu}] U_{\text{PMNS}}^\dagger / \langle H \rangle. \quad (2.5)$$

The matrix R drops out in the dim-5 coefficient matrix g_ν in (2.4), but will contribute to operators of dim-6 or higher. If $\Lambda_L \sim M_{\text{GUT}}$, the coefficients of the latter will be highly suppressed, while generic L -conserving NP effects, associated with a scale $\Lambda_{\text{LFV}} \gtrsim 1$ TeV, would clearly dominate.

2.2. Experimental situation

The experimental determination of neutrino-mixing parameters (see e.g. [6, 5] and references therein) reveals:

- two distinct mass-squared differences, $|\Delta m_{31}^2| \sim 2 \cdot 10^{-3} \text{ eV}^2$ related to atmospheric neutrino oscillations, and $\Delta m_{21}^2 \sim 7 \cdot 10^{-5} \text{ eV}^2$ to solar neutrino oscillations;

- a mixing angle, $\sin^2 \theta_{23} \simeq 0.5$, close to being maximal; a large mixing angle $\sin^2 \theta_{12} \sim 0.3$; and a small mixing angle θ_{13} .

Current data still leave open whether neutrinos have a normal/inverted hierarchical or a degenerate spectrum. The Majorana nature has to be verified/falsified, for instance by searching for ν -less double β -decay. Together with constraints from cosmology and from the endpoint of β -decay energy spectra, this also helps to set the absolute neutrino mass scale. A precise measurement of θ_{13} is foreseen in the near future, whereas the determination of CP-phases will be difficult (see e.g. the discussion in [10]).

2.3. Origin of flavour hierarchies?

The specific patterns of hierarchies in fermion masses and mixings observed in the lepton (and quark) flavour sector suggest a theoretical explanation in terms of additional (flavour) symmetries. A well-known example is the Froggatt-Nielsen approach [11], which postulates an additional (spontaneously broken) $U(1)$ symmetry together with heavy fermionic messenger fields and a particular choice of $U(1)$ charges for the different families. Other popular approaches are discrete (non-abelian) flavour symmetries (see e.g. [12]), which lead to particular textures in the fermion Yukawa matrices, or models with extra spatial dimensions (ED), where the different fermion families are displaced along the extra dimension and the mass hierarchies are explained by the imperfect overlap of the corresponding wave function profiles. For an overview of different models, see [5] and references therein.

3. Lepton-flavour violation

As already mentioned, LFV decays provide particularly sensitive probes of NP at the TeV scale, and the predicted correlations between various observables may help to distinguish different models, like SUSY, ED, Little Higgs ... LFV observables may be classified as follows: (i) dipole transitions, which induce the decays $\mu \rightarrow e\gamma$, $\tau \rightarrow \mu(e)\gamma$; (ii) 4-lepton transitions, $\mu \rightarrow 3e$, $\tau \rightarrow 3\mu$ etc., which also receive contributions from dipole operators via virtual photons; (iii) transitions involving 2 leptons and 2 quarks, which induce LFV hadronic decays, and ℓ - ℓ' conversion in nuclei.¹

While dipole transitions provide direct constraints on the coefficients of the corresponding operators $\ell\sigma^{\mu\nu}\ell'F_{\mu\nu}$, the analysis of 3-body leptonic decays is complicated by the fact that different chiralities in 4-lepton operators and the virtual photon contributions from dipole operators lead to

¹ Of similar importance for the phenomenological analysis of NP models are flavour-diagonal transitions, like the anomalous magnetic moment of the muon, $(g-2)_\mu$, or lepton electric dipole moments (EDMs).

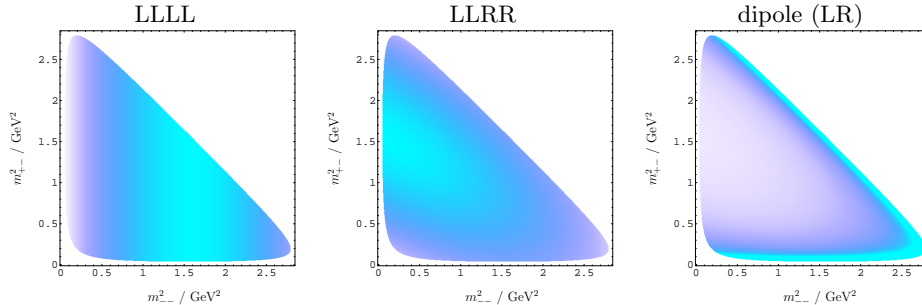


Fig. 1. Examples for phase space distributions in $\tau \rightarrow 3\mu$ induced by 4-lepton operators of different chirality (LLLL, LLRR), or by virtual photons from dipole operators (LR) [13]. In the Dalitz plots the vertical (horizontal) axis refers to the invariant mass of a muon pair of opposite (same) charge.

different phase-space distributions [13]. Depending on the dominance of one or the other operator, different regions in the Dalitz plot show different sensitivity on NP parameters (examples are shown in Fig. 1). This has to be taken into account for a model-independent analysis of experimental data. Finally, LFV transitions of class (iii) require hadronic matrix elements as non-perturbative input. In the following, we will present some illustrative examples based on specific NP predictions, and also comment on model-independent approaches based on minimal-flavour violation (MFV).

3.1. LFV phenomenology in specific models

SUSY extensions of the SM generically provide many new sources for LFV and CP-violation, and the phenomenological consequences have been extensively studied in the literature. Depending on see-saw parameters and on the (small) misalignment between fermions and sfermions in the soft SUSY-breaking sector, one can study correlations between different LFV decays. In a generic SUSY see-saw framework, one can derive inequalities like [14]

$$\text{BR}[\mu \rightarrow e\gamma] \gtrsim C \times \text{BR}[\tau \rightarrow \mu\gamma] \text{BR}[\tau \rightarrow e\gamma], \quad (3.1)$$

where C is a constant that can be calculated for a given set of SUSY parameters. More explicit correlations, e.g. between $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ [15], can only be obtained by specifying additional assumptions about (otherwise unobservable) Majorana neutrino parameters. Furthermore, depending on the SUSY parameters, LFV decays can be either dominated by gaugino-mediated or by Higgs-mediated flavour transitions, where the latter arise

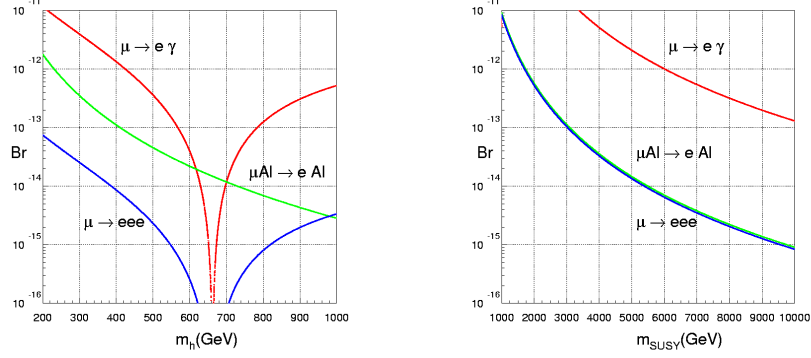


Fig. 2. Branching ratios of $\mu \rightarrow e\gamma$, $\mu \rightarrow eee$ and $\mu\text{Al} \rightarrow e\text{Al}$. Left: For Higgs-mediated LFV as a function of the Higgs boson mass m_h . Right: For gaugino-mediated LFV as a function of a common SUSY mass m_{SUSY} . In both cases $\tan\beta = 50$ and $\delta_{LL}^{21} = 10^{-2}$. Figures taken from [16].

from loop-induced non-holomorphic Higgs couplings.² In the two cases, the observables considered in Fig. 2 show quite different correlations. Finally, interesting correlations between $\mu \rightarrow e\gamma$ and flavour-diagonal observables ($(g-2)_\mu$ and leptonic EDMs) have been explored in [17].

Another class of SM extensions with interesting LFV effects are “Littlest Higgs Models” with T -parity (LHT). Besides new gauge bosons (which can be detected at the LHC), these models contain new doublets of mirror leptons (and quarks) with masses of order TeV, which may induce LFV rates that exceeds the SM case by orders of magnitude. Phenomenological predictions depend on the LHT scale parameter, f , the masses of the mirror leptons, $M_{H_i}^\ell$, the 3 mixing angles among the mirror leptons θ_{ij}^ℓ and 3 new (Dirac) phases δ_{ij}^ℓ . In Fig. 3 we show as an example the correlation between $\mathcal{B}[\mu \rightarrow 3e]$ and $\mathcal{B}[\mu \rightarrow e\gamma]$ for $f = 1$ TeV, and $300 \text{ GeV} \leq M_{H_i}^\ell \leq 1.5 \text{ TeV}$ as calculated in [18]. Two important conclusions can be drawn: First, the mirror leptons must be quasi-degenerate and/or the mixings have to be very hierarchical in order to fulfill the present bounds on the individual decays. Second, the considered LHT scenarios, where 3-lepton decays (and also μ - e conversion) are dominated by Z^0 -penguin and box diagrams, can be clearly distinguished from the MSSM, where dipole operators (or Higgs-boson induced effects) play the dominant role. For a comparison, see Table 2 [18] (for a discussion of LFV effects in minimal see-saw models, see [19]).

² Notice that the possible strength of Higgs-induced flavour transitions in SUSY is also constrained from quark decays measured e.g. in $B \rightarrow \tau\nu_\tau$ or $B \rightarrow X_s\gamma$.

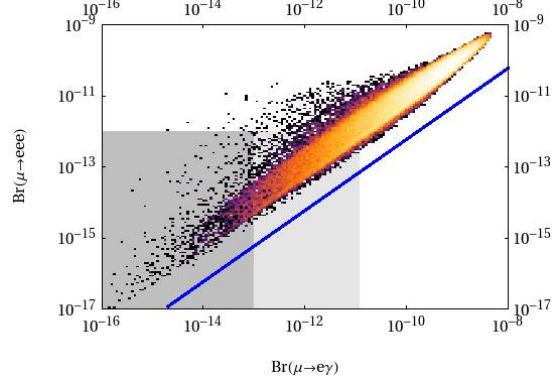


Fig. 3. $\mathcal{B}[\mu \rightarrow 3e]$ vs. $\mathcal{B}[\mu \rightarrow e\gamma]$ in LHT for $f = 1$ TeV and $300 \text{ GeV} \leq M_{H_i}^\ell \leq 1.5 \text{ TeV}$ from [18]. The lower line denotes the contribution from the dipole operator via $\mu \rightarrow e\gamma^*$, and the light (dark) grey areas the present (foreseen) exp. bounds.

Table 2. Comparison of LFV transitions in LHT and MSSM: Branching fractions for 3-lepton decays and μ - e conversion, relative to the corresponding radiative decays (from [18]).

ratio	LHT	MSSM (dipole)	MSSM (Higgs)
$\frac{Br(\mu^- \rightarrow e^- e^+ e^-)}{Br(\mu \rightarrow e\gamma)}$	0.02...1	$\sim 6 \cdot 10^{-3}$	$\sim 6 \cdot 10^{-3}$
$\frac{Br(\tau^- \rightarrow e^- e^+ e^-)}{Br(\tau \rightarrow e\gamma)}$	0.04...0.4	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$
$\frac{Br(\tau^- \rightarrow \mu^- \mu^+ \mu^-)}{Br(\tau \rightarrow \mu\gamma)}$	0.04...0.4	$\sim 2 \cdot 10^{-3}$	0.06...0.1
$\frac{Br(\tau^- \rightarrow e^- \mu^+ \mu^-)}{Br(\tau \rightarrow e\gamma)}$	0.04...0.3	$\sim 2 \cdot 10^{-3}$	0.02...0.04
$\frac{Br(\tau^- \rightarrow \mu^- e^+ e^-)}{Br(\tau \rightarrow \mu\gamma)}$	0.04...0.3	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$
$\frac{Br(\tau^- \rightarrow e^- e^+ e^-)}{Br(\tau \rightarrow e\gamma)}$	0.8...2.0	~ 5	0.3...0.5
$\frac{Br(\tau^- \rightarrow e^- \mu^+ \mu^-)}{Br(\tau \rightarrow e\gamma)}$	0.7...1.6	~ 0.2	5...10
$\frac{R(\mu\text{Ti} \rightarrow e\text{Ti})}{Br(\mu \rightarrow e\gamma)}$	$10^{-3} \dots 10^2$	$\sim 5 \cdot 10^{-3}$	0.08...0.15

3.2. Minimal flavour violation

As already mentioned, the non-observation of LFV decays puts severe constraints on NP parameters. For instance, allowing for generic coupling constants in front of effective operators, the bound on $\mu \rightarrow e\gamma$ would already set a lower bound on the NP scale, $\Lambda_{\text{LFV}} > 10^5 \text{ TeV}$. In order to avoid ad-hoc fine-tuning of parameters, one could introduce the concept

Table 3. Partial breaking of the lepton-flavour symmetry group by the atmospheric mass squared difference for normal or inverted hierarchy [22].

hierarchy	symmetry	approx. spurion VEV
normal	$SU(3)_L \times U(1)_L$ $\rightarrow U(2)_L \times Z_2$	$\langle g_\nu \rangle \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \frac{\Lambda_L \sqrt{\Delta m_{\text{atm}}^2}}{v^2}$
inverted	$SU(3)_L \times U(1)_L$ $\rightarrow SO(2)_L \times U(1)_{L_3}$	$\langle g_\nu \rangle \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \frac{\Lambda_L \sqrt{\Delta m_{\text{atm}}^2}}{v^2}$

of minimal flavour violation in the lepton sector (MLFV [20, 21]). Using an effective-theory approach, one relates the NP flavour coefficients to the SM ones by considering the flavour matrices as vacuum expectation values (VEVs) of spurion fields. Compared to the quark sector, additional complications arise, because the neutrino masses themselves already imply a (model-dependent) extension of the SM. Therefore the specification of the neutrino field content and the mechanism for the generation of neutrino masses should be considered part of the effective-theory construction, on top of the MFV hypothesis. In particular, the scale for lepton-*number* violation Λ_L should be distinguished from the LFV scale, Λ_{LFV} .

Focusing on the minimal extension of the SM, the flavour group in the lepton sector is given by $U(3)_L \times U(3)_{E_R}$, i.e. independent unitary transformations of left-handed doublets and right-handed singlets. The flavour symmetry is broken by the charged-lepton Yukawa matrices and the flavour matrix in front of the effective dim-5 operator (2.2).

$$\text{Yukawa: } Y_E \sim (3, \bar{3})_{1,-1}, \quad \text{Dim-5: } g_\nu \sim (\bar{6}, 1)_{2,0}. \quad (3.2)$$

In the mass eigenbasis for charged leptons, the matrix g_ν can be expressed in terms of the neutrino masses and the PMNS mixing matrix,

$$g_\nu = \frac{\Lambda_L}{v^2} U_{\text{PMNS}}^* \text{diag}[m_\nu] U_{\text{PMNS}}^\dagger. \quad (3.3)$$

In MLFV, effective 2- and 4-lepton operators for flavour transitions are now constructed in such a way that they are formally invariant under the flavour group (and the SM gauge group), which is achieved by inserting appropriate powers of g_ν and Y_E with overall (flavour-independent) coupling constants of order 1. A complete set of operators can be found in [20]. The leading effect due to large mass-squared difference Δm_{atm}^2 observed in atmospheric

neutrino oscillations can be singled out by using a non-linear representation of MLFV [22]. Depending on the neutrino mass hierarchy, this also implies a particular (partial) breaking of the lepton flavour symmetry,³ see Table 3. The solar mass differences, as well as the mixing parameters in U_{PMNS} are then associated to spurion fields of the residual flavour symmetry.

4. Summary

³ This can be viewed as the first step of a sequence of flavour symmetry breaking, as discussed for the quark sector in [23].

can thus be viewed as complementary to precision measurements of the CKM parameters and rare quark decays and to the direct search for new particles and interactions at hadron colliders in the LHC era.

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